Limits on Static Shape Control for Space Structures

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This paper deals with correction of shape distortion due to zero-mean normally distributed errors in structural sizes. The concept of ideal actuators—actuators that can produce any desired displacement field—is introduced. Using this concept, a bound on the possible improvement in the expected value of the root-mean-square shape error is obtained. The shape correction associated with the ideal actuators is also characterized. An actuator effectiveness index is developed by comparing the displacement field generated by the actuator to the ideal. The results are specialized to a simple form for truss structures composed of nominally identical members. The bound and effectiveness index are tested on a 55-m radiometer antenna truss structure. It is found that previously obtained results for optimum actuators had a performance close to the bound obtained here. Also, it is found that large numbers of actuators are needed for large reductions in shape errors. Furthermore, the actuators associated with the optimum design are shown to have high effectiveness indices. Since only a small fraction of truss elements tend to have high effectiveness indices, the use of the effectiveness index can greatly reduce the number of truss members that need to be considered as actuator sites.

Introduction

MANUFACTURING errors are recognized as a major cause of shape distortion in large space structures.[1] These errors, as well as many other sources of distortion (such as variations in the coefficient of thermal expansion), are random in nature, and a statistical analysis needs to be carried out to estimate the resulting shape distortion.[1-3] When actuators are used to reduce shape distortion, the corrected shape is also random in nature and requires a more complex and costly statistical analysis.[3]

Trusses are one candidate for large space structures. When these trusses are used as backup structures for electromagnetic reflectors, maintaining accurate surface shape is important. Optimizing locations of actuators used to reduce shape distortion on large trusses is a formidable problem because of the discrete nature of the problem. The actuators likely to be used in large space structure are located in truss elements and control their length (e.g., piezoelectric actuators). Given m truss elements and n actuators we have $\binom{n}{m}$ possibilities of locating them. Several ad hoc methods for tackling this combinatorial problem were proposed.[4-6] However, these methods are not guaranteed to converge even to a local optimum, and therefore the quality of the actuator configuration they produce is difficult to evaluate.

The present paper addresses this difficulty by exploring the shape correction possible with ideal actuators. These ideal actuators are assumed to be able to generate any correction field desired by the designer. However, physical actuators cannot be moved around in the structure in response to different shape distortion fields, and only the amplitude of their action can be changed. To mirror this situation it is assumed that the ideal actuator correction field has to be selected before the random errors are known, with only its amplitude selected by a control system in orbit. The task of the designer is defined here to select the correction fields of these ideal actuators so that the expected value of the error after correction is minimal. The expected value is calculated based on the assumption of uncorrelated normally distributed error sources.

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The concept of ideal actuators is used to obtain two useful results. The first is a lower bound on the possible reduction in shape error with n actuators. This result can be used to estimate the number of actuators needed for a particular application. The second result is a characterization of the goodness of a physical actuator in terms of an effectiveness index that measures the closeness of its correction field to the ideal.

A key question is how useful are results obtained with ideal actuators to the real-life case. An attempt to answer this question is presented by analyzing a 55-m antenna truss for which actuator locations were optimized in Ref. 3. It is shown that the expected values of the corrected root-mean-square error are reasonably close to the bound obtained here. Furthermore, it is shown that the optimization procedure used in Ref. 3 has picked, indeed, actuators with correction fields that had very high effectiveness indices. Unfortunately, it is also shown that for an order of magnitude reduction in shape error we need a very large number of actuators.

It is assumed in this paper that sensors are available to correctly measure the distortion field. This is a major assumption as on-orbit measurement of surface distortion may not be currently feasible. Errors in sensor readings can substantially degrade the capability to correct surface distortion.[7] However, this problem is beyond the scope of the present paper.

Linear Distortion Correction

The structure is assumed to have a set of n_s sensors that measure n_s components of the distortion field, and a set of n actuators used to correct the distortion. Denoting the vector of sensor-measured components of the distortion as $\hat{\psi}$, we assume that the actuators seek to minimize a weighted root-mean-square (rms) measure of the distortion ψ_{rms} defined as

$$\psi_{\text{rms}}^2 = \hat{\psi}^T B \hat{\psi} \tag{1}$$

where B is a positive semidefinite weighting matrix.

It is convenient to work in a transformed (modal) coordinate system where the matrix B is the unit matrix. That is, we write $\hat{\psi}$ as a linear combination of modes ϕ_i $i = 1, \ldots, n_s$

$$\hat{\psi} = \Phi\psi \tag{2}$$

where the matrix Φ is composed of columns ϕ_i which are orthonormal with respect to B. That is

$$\Phi^T B \Phi = I \tag{3}$$

where I denotes the unit matrix. Then from Eqs. (1–3)

$$\psi_{\rm rms}^2 = \psi^T\psi \tag{4}$$

The design of a system of actuators for reducing the shape error can be separated into two steps. In the first step the nature and location of the actuators is determined. Then, in the second step, for a given distortion field, the amplitude of each actuator correction field must be determined so as to reduce the distortion. The first step involves choice of hardware and its location in the structure, and it is assumed here that this choice is made before the distortion is known. The second step is performed in response to a measured distortion, and requires a simple computational procedure described below.

We assume that the behavior of the structure and actuators is linear, so that if a unit action of the ith actuator produces a displacement vector u_i, then the corrected shape vector δ is given as

$$\delta = \psi + \sum_{i=1}^{n} u_i\theta_i = \psi + U\theta \tag{5}$$

where θ is a vector of actuator outputs (with components θ_i) and U is a matrix with columns u_i. Note that the components of all of these vectors are modal (amplitude) coordinates with the physical vector (denoted with a hat) obtained by the modal transformation, e.g.,

$$\hat{u}_i = \Phi u_i \tag{6}$$

The rms of the corrected distortion is

$$\delta_{\rm rms}^2 = \hat{\delta}^T B \hat{\delta} = \delta^T\delta \tag{7}$$

The optimum vector θ which minimizes $\delta_{\rm rms}$ is easily shown (e.g., Ref. 4) to be the solution of the system

$$A\theta = r \tag{8}$$

where

$$A = U^TU \tag{9a}$$

$$r = -U^T\psi \tag{9b}$$

and then the corrected shape δ is

$$\delta = \psi - UA^{-1}U^T\psi = (I - UA^{-1}U^T)\psi = G\psi \tag{10}$$

and

$$\delta_{\rm rms}^2 = \psi_{\rm rms}^2 - \psi^TUA^{-1}U^T\psi \tag{11}$$

The calculation of θ from Eq. (8) is the only computation that needs to be performed on orbit. With the matrix A factored ahead of time, this requires about n^2 floating point operations.

Statistical Analysis

Let the distortion field be due to a set of errors of disturbances in the structure characterized by their amplitude parameters ε_i $i = 1, \ldots, N$. Often the statistical properties of the ε_i are known and we want to obtain the statistics of ψ and δ. Since the behavior of the structure is linear the total distortion due to the N errors, ψ, is given as

$$\psi = \sum_{i=1}^{N} \varepsilon_i\gamma_i = \Gamma e \tag{12}$$

where γ_i is the shape distortion due to a unit ε_i, Γ is a matrix with γ_i as its ith column, and e is the vector of ε_is.

We assume that the ε_i are zero-mean, normally distributed random variables with a covariance matrix Σ

$$\Sigma = E(ee^T) \tag{13}$$

where E denotes the expected value. The diagonal terms of Σ are the squares of the standard deviations of the ε_i, while the off-diagonal terms measure the correlation between them. Because ψ is a linear combination of all ε_i it is also a normally distributed zero-mean random variable, and all its statistical properties are summarized in its covariance matrix $C_{\psi\psi}$. In particular, the diagonal terms of $C_{\psi\psi}$ are the standard deviations of the modal amplitudes of the distortion. From Eq. (12) the covariance matrix of ψ can be calculated as

$$C_{\psi\psi} = E(\psi\psi^T) = \Gamma E(ee^T)\Gamma = \Gamma\Sigma\Gamma^T \tag{14}$$

Using Eq. (4) we get that the expected value of $\psi_{\rm rms}^2$ is

$$E(\psi_{\rm rms}^2) = \sum_{i=1}^{ns} (C_{\psi\psi})_{ii} \tag{15}$$

that is the trace of $C_{\psi\psi}$. The expected value of $\delta_{\rm rms}^2$ is obtained by rewriting Eq. (11) as

$$\delta_{\rm rms}^2 = \psi_{\rm rms}^2 - W^TW \tag{16}$$

where

$$W = L^TU^T\psi \tag{17}$$

and L is a square factor of A^{-1} ($A^{-1} = LL^T$, e.g., the Cholesky factor). The covariance matrix of W is then given as

$$C_{WW} = L^TU^TC_{\psi\psi}UL \tag{18}$$

$$E(\delta_{\rm rms}^2) = \sum_{i=1}^{ns} (C_{\psi\psi})_{ii} - \sum_{i=1}^{n} (C_{WW})_{ii} \tag{19}$$

Appendix A provides also the standard deviations of $\psi_{\rm rms}^2$ and $\delta_{\rm rms}^2$. The effectiveness of the actuators is measured by the distortion ratio, g, defined as

$$g^2 = \frac{E(\delta_{\rm rms}^2)}{E(\psi_{\rm rms}^2)} \tag{20}$$

Ideal Actuators

The problem of optimizing the properties of the actuators (such as location) requires the minimization of $E(\delta_{\rm rms}^2)$ which is the same as the maximization of the trace of C_{WW}. The choice of actuator properties, such as location, affects the matrix U of actuator correction vectors. We introduce the concept of ideal actuators as actuators that can generate the most effective set of correction vectors. In Appendix B it is shown that such ideal actuators produce vectors u_i (columns of U) that are linear combinations of the eigenvectors of $C_{\psi\psi}$ associated with its nth highest eigenvalues. Any such set of linear combination of eigenvectors is fine, as long as these vectors are linearly independent (if the vectors are close to being linearly dependent then the components of the actuator action vector θ can become very large, see Ref. 8). If we arrange the eigenvalues of $C_{\psi\psi}$ in ascending order

$$0 \leq \lambda_1 \leq \lambda_2 \cdots \leq \lambda_{ns} \tag{21}$$

then we get that the ideal value of the corrected rms is given as the sum of the lower eigenvalues of $C_{\psi\psi}$

$$E(\delta^2_{\rm rms})_{\rm ideal} = \sum_{i=1}^{ns-n} \lambda_i \tag{22}$$

Note that the expected value of the $\psi^2_{\rm rms}$ was the trace of $C_{\psi\psi}$ which is equal to the sum of all its eigenvalues. With n ideal actuators we can remove the n largest eigenvalues from that sum!

For most problems it will be impossible to select actuators that produce displacement fields that are exact linear combinations of the eigenvectors v_j corresponding to the largest eigenvalues of $C_{\psi\psi}$. However, it may be possible to come close. In particular, we can judge the suitability of an actuator by checking how close is its displacement field to such a linear combination. An effectiveness index proposed here is the cosine of the angle between the displacement field produced by an actuator and its projection on the subspace spanned by the eigenvectors v_j. If the eigenvectors v_j of $C_{\psi\psi}$ are normalized to unit length then they are an orthonormal set

$$v_i^T v_j = \delta_{ij} \tag{23}$$

and we can use them as a basis for expressing the vector u_i as

$$u_i = \sum_{j=1}^{ns} (v_j^T u_i) v_j \tag{24}$$

Then the projection of u_i on the subspace spanned by the eigenvectors corresponding to the n largest eigenvalues is given by the last n terms in Eq. (24). The cosine of the angle between that projection and u_i is denoted as q_i and is given as

$$q_i = \left(\sum_{j=ns-n+1}^{ns} (v_j^T u_i)^2 \right)^{1/2} \Big/ (u_i^T u_i)^{1/2} \tag{25}$$

Only actuators with an effectiveness index close to one are likely to be good actuators. The calculation of the effectiveness index requires the calculation of $C_{\psi\psi}$ and its eigenvectors. This in turn requires the calculation of the matrix Γ. While these computations are not trivial, they do not have to be performed on orbit, and so they represent a few seconds of a supercomputer CPU time.

Applications to Truss Structures

Consider a truss structure composed of a large number (N) of members of similar lengths, and where the shape distortion is due mostly to length errors (typically manufacturing errors). We assume that the length errors are uncorrelated, and that the standard deviation of the ith member is Δl_i so that the covariance matrix Σ is

$$\Sigma = \mathrm{diag}(\Delta l_i^2) \tag{26}$$

The sensed components are often the vertical displacements of the surface of the truss to which the antenna is attached, and the matrix B is a diagonal matrix with $1/n_s$ on the diagonal.

Any set of modes that satisfy the orthonormality condition, Eq. (3), was adequate for the analysis up to now. However, Hedgepeth[1] showed that particularly simple results may be obtained if we use a special set of vibration modes obtained by treating B as a mass matrix. To obtain these modes, ϕ_j, we expand the matrix B to the full set of finite element degrees of freedom by adding rows and columns of zeroes, denote it as $\overline{B}$, treat it as a mass matrix, and solve for the vibration modes from

$$(\overline{K} - \omega_j^2 \overline{B})\overline{\phi}_j = 0 \tag{27}$$

where $\overline{K}$ is the stiffness matrix, and $\overline{\phi}_j$ denotes the jth B-vibration mode (excluding rigid body modes). The sensed components of $\overline{\phi}_j$ are extracted as

$$\hat{\phi}_j = T\overline{\phi}_j \tag{28}$$

where T is a Boolean matrix. Note that

$$B = T^T \overline{B} T \tag{29}$$

Because the rank of $\overline{B}$ is smaller than or equal to n_s, only n_s or less of the frequencies ω_j have finite values. The infinite frequencies correspond to zero-rms modes satisfying

$$\overline{\phi}^T \overline{B} \overline{\phi}_j = \hat{\phi}_j^T B \hat{\phi}_j = 0 \tag{30}$$

To calculate the distortion associated with unit length error in the ith member (corresponding to $\varepsilon_i = 1$) we apply a force vector F_i to the truss and solve

$$\overline{K}\overline{\gamma}_i = F_i \tag{31}$$

where F_i consists of a pair of forces colinear with the ith member of magnitude

$$F_i = (EA)_i / l_i \tag{32}$$

where $(EA)_i$ and l_i denote the axial rigidity and length, respectively, of the ith member. In Appendix C it is shown that the covariance matrix $C_{\psi\psi}$ can be calculated without solving Eq. (31) as

$$(C_{\psi\psi})_{j\ell} = \frac{1}{\omega_j^2 \omega_\ell^2} \sum_{i=1}^{N} \Delta l_i^2 (\overline{\phi}_j^T F_i)(\overline{\phi}_\ell^T F_i) \tag{33}$$

The trace of $C_{\psi\psi}$ is equal to $E(\psi^2_{\rm rms})$, while the ideal $E(\delta^2_{\rm rms})$ is given in terms of the eigenvalues of the matrix by Eq. (22). We can then calculate the ideal distortion ratio $(g^2)_{\rm ideal}$ from Eq. (20). Hedgepeth[1] showed that further simplification is possible if

$$\Delta l_i^2 (EA)_i / l_i = c \tag{34}$$

Then (see Appendix C)

$$(C_{\psi\psi})_{j\ell} = \frac{c}{\omega_\ell^2} \delta_{j\ell} \tag{35}$$

This happens, in particular, when all the members of the truss have identical nominal properties. In this case, since $C_{\psi\psi}$ is diagonal its eigenvalues are equal to the diagonal elements, and Eq. (22) becomes

$$E(\delta^2_{\rm rms})_{\rm ideal} = \sum_{j=1}^{ns-n} \lambda_j = c \sum_{j=n+1}^{ns} \frac{1}{\omega_j^2} \tag{36}$$

and the distortion ratio of Eq. (20) becomes

$$(g^2)_{\rm ideal} = \frac{\displaystyle\sum_{j=n+1}^{ns} \frac{1}{\omega_j^2}}{\displaystyle\sum_{j=1}^{ns} \frac{1}{\omega_j^2}} \tag{37}$$

Note that in this case the eigenvectors of $C_{\psi\psi}$ can be taken to be the unit coordinate vectors. That is, for the case of a truss satisfying Eq. (34) the ideal actuators produce a deformation state that is a linear combination of the first n B-vibration modes. These actuators eliminate the part of the distortion which is in the subspace spanned by these modes. The corrected shape δ is then equal to ψ with its first n components zeroed out.

Application to 55-m Antenna Truss

The truss support structure for a 55-m radiometer antenna shown in Fig. 1 was used as an example. The tetrahedral truss consists of 420 members connected at 109 joints. The rms of the vertical motion of the upper surface points was used as a measure of the error. This corresponds to the vector $\hat{\psi}$ consisting of the vertical displacements of the 61 joints on the upper surface, and the matrix B equal to $(1/\sqrt{61})I$.

The truss elements of the upper and lower surfaces have similar lengths of about 315 in. with nominal length variations of about 1%. The diagonal elements connecting the lower and upper surfaces are about 180 in. long with nominal length variations of about 3–4%. This antenna structure was analyzed in Ref. 3 assuming that shape errors are introduced due to manufacturing length errors from the nominal values. The length errors were assumed to be uncorrelated and of equal standard deviation (all Δl_i equal). The actuators were assumed to be mechanisms (such as screws) embedded in some elements that can be used to change the length of these elements.

Two methods were used to obtain optimum actuator positions on the upper surface of the antenna in Ref. 3. First an approximate procedure that permitted actuators to be located anywhere in space (that is, not restricted to truss member location) was used. This procedure employed the conjugate gradient method to minimize $E(\delta^2_{\rm rms})$. The second procedure was a heuristic integer programming approach called ESPS (exhaustive single-point substitution[4]).

The optimum actuator locations for the cases of three and six actuators are shown in Figs. 2 and 3, respectively. The corresponding optimal values of the distortion ratio g, Eq. (12), are compared in Table 1 with the lower bound on g predicted by Eq. (37). It is seen that the approximate optimum (which permits actuators even where there are no truss elements) comes close to the lower bound based on the assumption of ideal actuators. The ESPS optimum was calculated for up to 18 actuators and is seen to lag not too far behind the bound. This good agreement is achieved in spite of the fact that the assumption of Eq. (34) is not met exactly

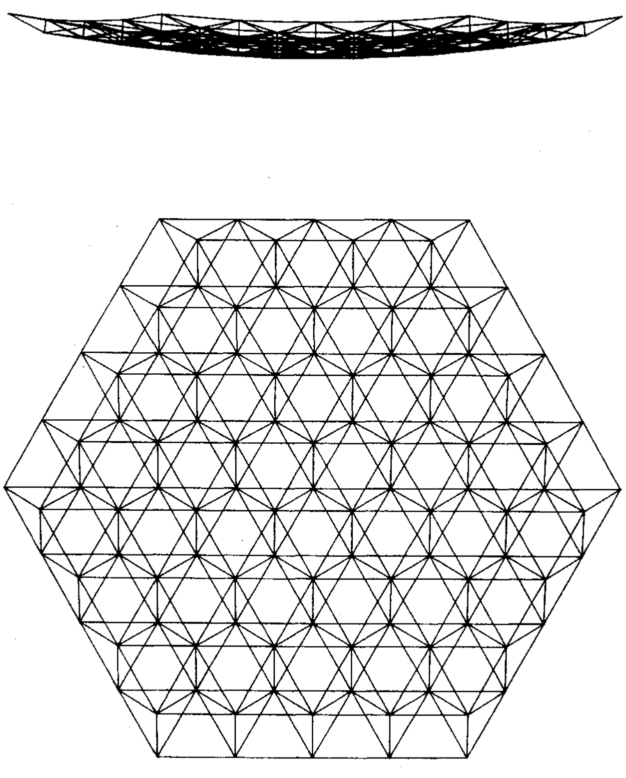

Fig. 1 Side and top views of tetrahedral truss antenna reflector.

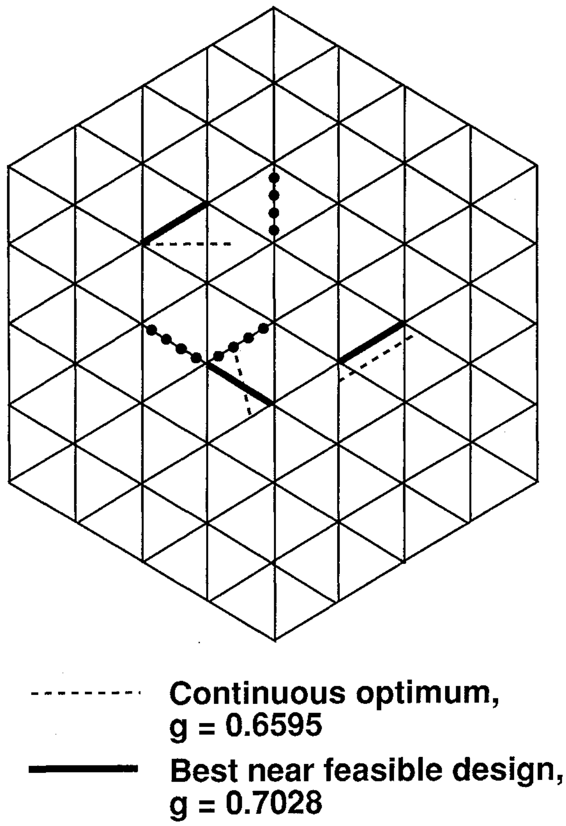

Fig. 2 Optimum upper surface actuator configurations—three actuators. ---, Continuous optimum, $g = 0.6595$; ——, best near feasible design, $g = 0.7028$; •••, ESPS solution, $g = 0.6919$.

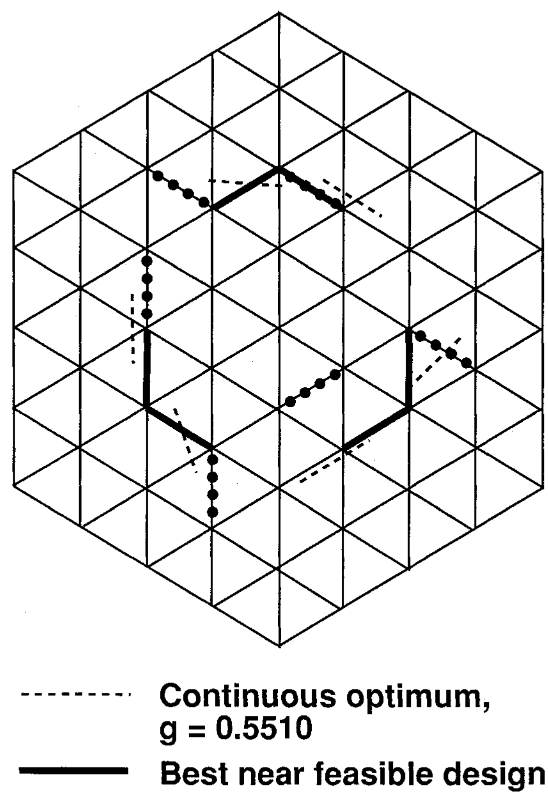

Fig. 3 Optimum upper surface actuator configurations—six actuators. ---, Continuous optimum, $g = 0.5510$; ——, best near feasible design, $g = 0.6039$; •••, ESPS solution, $g = 0.5967$.

Table 1 Effect of number of actuators on optimally corrected surface error

	Distortion ratio g			
Number of actuators	Lower bound Eq. (37)	ESPS optimum	Approximate optimum (Ref. 3)	$\sigma(\delta^2_{\rm rms})/E(\delta^2_{\rm rms})$
3	0.6384	0.6919	0.6595	0.2324
6	0.5346	0.5967	0.5510	0.2166
9	0.4617	0.5271		0.2074
12	0.4049	0.4710		0.2066
15	0.3531	0.4061		0.2017
18	0.3044	0.3574		0.1830
24	0.2486			0.1823
30	0.2099			0.1951
36	0.1756			0.2157
42	0.1438			0.2509
48	0.1103			0.3165
54	0.0683			0.5002

(with small variations for surface elements and a large discrepancy for the diagonal elements). The agreement indicates that the lower bound based on ideal actuators may be a good way for estimating the shape controllability of an antenna structure.

Table 1 also contains distortion ratios for even larger number of actuators (up to 54). Because the rms is calculated based on 61 sensed components, 58 linearly independent actuators will always reduce any shape error to zero (the number of 58 is obtained by noting that the shape error is orthogonal to three rigid body modes). Table 1 indicates that large reductions in shape error require close to 58 actuators. This is a disheartening result, especially in view of the fact that even if we zero out all the sensed components of the error, other components may not be zero. The last column in Table 1 is the coefficient of variation of $\delta^2_{\rm rms}$. Except for very small values of the distortion ratio, the coefficient of variation is around 0.2. This indicates that the actual corrected rms error will be usually close to its expected value.

Since large reductions in the shape error appear to require a large number of actuators, one may ask whether other means are available to reduce shape errors. Here the possibility of redesigning the truss by changing the cross-sectional areas of the members was considered. It was found that large changes in cross-sectional areas of individual members or groups of members had only a negligible effect on the expected value of the shape errors (original or corrected). Changes of 10% in the cross-sectional area of six members typically produced a 10^{-5} relative change in the error. Next, the possibility of removing some members (since the antenna is statically indeterminate) was considered. However, this would lower the stiffness of the truss without affecting the "mass" (B matrix) associated with the B-vibration modes. Thus it would lower the B frequencies and thus result in larger errors. Finally, changing the distance between the two surfaces of the antenna was considered. However, even with the assumption that an increase in the length of diagonal members will not increase the errors in these members, the effect was not large. Doubling the distance between the surfaces reduced the error by about a quarter.

A check on the effectiveness index of the individual actuators used in the ESPS solution was performed next. For the three-actuator case, q_i ranged between 0.130 and 0.974 for the 420 elements of the truss. The elements selected by the ESPS procedure as actuators, on the other hand, had effectiveness indices q_i between 0.961 and 0.974, indicating that they produce a displacement field overwhelmingly represented by the highest three modes of $C_{\psi\psi}$. It is interesting to note that only 29 of the 420 members had q_i larger than 0.95, so that the selection of actuator locations could have been greatly simplified. For the six-actuator case, the subspace spanned by six modes is larger, and q_i ranged between 0.164 and 0.980, with the ESPS procedure selecting actuators in the range 0.966 to 0.980. Of the 420 members, 36 had q_i larger than 0.960 and 45 had q_i larger than 0.95. It should be noted that it is not reasonable to simply select the actuators with the highest effectiveness indices. It is important that the actuators produce displacement field that are not linearly dependent or close to being linearly dependent.[8] These results show that the effectiveness index of Eq. (30) can be used to screen actuator locations and greatly reduce the number of actuator locations that need to be considered in the search for the optimum locations. For example, for the six-actuator case, if we consider only the 45 members with q_i greater than 0.95, the number of possible configurations drops from 7×10^{12} to 12×10^6.

Concluding Remarks

This work investigated the correction of shape distortion due to errors in structural sizes that are zero-mean normally distributed random variables. A bound on the maximum improvement in the expected value of the root-mean-square shape error was obtained as well as an effectiveness index of a given actuator. The results were specialized to a simple form for truss structures composed of nominally identical members. The bound was tested on a 55-m radiometer antenna truss structure. It was found that previously obtained results for optimum actuators had a performance close to the bound obtained here. Unfortunately, the results obtained with the bound indicate that a large number of actuators is needed for significant reductions in shape errors. It was also verified that the optimum actuators found previously rank very high in terms of the effectiveness index developed here. The effectiveness index can therefore be used to screen potential actuator locations and reduce drastically the number of truss members that need to be considered in the selection for actuator placement.

Appendix A: Standard Deviations of $\psi^2_{\rm rms}$ and $\delta^2_{\rm rms}$

From Ref. 9 we have that Eq. (4) implies that the variance of $\psi^2_{\rm rms}$ is

$$\sigma^2(\psi^2_{\rm rms}) = 2\sum_{i=1}^{ns}\sum_{j=1}^{ns}(C_{\psi\psi})^2_{ij} \tag{A1}$$

The variance of $\delta^2_{\rm rms}$ is obtained by first using Eq. (10) to obtain the covariance matrix of δ as

$$C_{\delta\delta} = GC_{\psi\psi}G^T \tag{A2}$$

and then in analogy with Eq. (A1)

$$\sigma^2(\delta^2_{\rm rms}) = 2\sum_{i=1}^{ns}\sum_{j=1}^{ns}(C_{\delta\delta})^2_{ij} \tag{A3}$$

Appendix B: Derivation of Ideal Actuators

We rewrite Eq. (18) as

$$C_{WW} = U^{*T} C_{\psi\psi} U^* \tag{B1}$$

where

$$U^* = UL \tag{B2}$$

First we note that U^* consists of a set of orthonormal vectors. Indeed

$$(U^{*T}U^*)^{-1} = (L^T U^T U L)^{-1} = (L^T A L)^{-1}$$
$$= L^{-1}A^{-1}L^{-T} = L^{-1}LL^TL^{-T} = I \tag{B3}$$

It is shown in Ref. 10 (theorem 1) that

$$\max_{U^*} \sum_i (C_{WW})_{ii} = \sum_{j=n_s-n+1}^{n_s} \lambda_j \tag{B4}$$

where λ_j are the eigenvalues of $C_{\psi\psi}$ arranged in ascending order. We are able to realize this maximum by taking the columns of U^* to be the eigenvectors v_j, $j = n_s - n + 1, \ldots, n$ corresponding to these largest eigenvalues. (From the derivation in Ref. 9 it is also clear that any linear combination of these eigenvectors will do.) From Eq. (B2) we see that this is equivalent to taking U as a set of linearly independent combinations of these eigenvectors.

Appendix C: Calculation of $C_{\psi\psi}$ for Truss Structures

Once we calculate $\bar{\gamma}_i$ from Eq. (31), we can then extract its sensed components as

$$\hat{\gamma}_i = T\bar{\gamma}_i \tag{C1}$$

The distortion vector due to the ith error source can be written in terms of modal coordinates

$$\bar{\gamma}_i = \sum_{j=1}^{ns} \gamma_{ij}\bar{\phi}_j \tag{C2}$$

where

$$\gamma_{ij} = \bar{\phi}_j^T \bar{B} \bar{\gamma}_i = \hat{\phi}_j^T B \hat{\gamma}_i \tag{C3}$$

Equation (C3) implies that the modal coordinates of $\bar{\gamma}_i$ and of $\hat{\gamma}_i$ are the same. This is a result of the fact that the modes beyond n_s do not contribute to the rms

$$(\gamma_i^2)_{\rm rms} = \hat{\gamma}_i^T B \hat{\gamma}_i = \bar{\gamma}_i^T \bar{B} \bar{\gamma}_i = \sum_{j=1}^{ns} \gamma_{ij}^2 \tag{C4}$$

The modal coordinates can be calculated without solving first Eq. (31) by premultiplying that equation by $\bar{\phi}_j^T$ and using Eq. (C2) to obtain

$$\bar{\phi}_j^T \bar{K} \sum_{\ell=1}^{ns} \gamma_{i\ell}\bar{\phi}_\ell = \bar{\phi}_j^T F_i \tag{C5}$$

But

$$\bar{\phi}_j^T \bar{K} \bar{\phi}_\ell = \omega_j^2 \delta_{jl} \tag{C6}$$

where $\delta_{j\ell}$ is the Kronecker delta. Altogether we get

$$\gamma_{ij} = \bar{\phi}_j^T F_i / \omega_j^2 \tag{C7}$$

We can simplify expressions by rewriting Eq. (C6) in terms of the element stiffness matrices instead of the global stiffness matrix as

$$\bar{\phi}_j^T \bar{K} \bar{\phi}_\ell = \sum_{i=1}^{N} \bar{\phi}_j^T K_i \bar{\phi}_\ell \tag{C8}$$

where K_i is the stiffness matrix of the ith element. Then

$$\bar{\phi}_j^T \bar{K} \bar{\phi}_\ell = \sum_{i=1}^{N} (\bar{\phi}_j^T F_i)(\bar{\phi}_\ell^T F_i) l_i/(EA)_i = \omega_j^2 \delta_{j\ell} \tag{C9}$$

Finally using Eqs. (14), (26), and (C7), we get

$$(C_{\psi\psi})_{j\ell} = \sum_{i=1}^{N} \Delta l_i^2 \gamma_{ij}\gamma_{i\ell} = \frac{1}{\omega_j^2 \omega_\ell^2} \sum_{i=1}^{N} \Delta l_i^2 (\bar{\phi}_j^T F_i)(\bar{\phi}_\ell^T F_i) \tag{C10}$$

If the Hedgepeth condition

$$\Delta l_i^2 (EA)_i / l_i = c \tag{34}$$

is satisfied, then a comparison of Eqs. (C9) and (C10) yields

$$(C_{\psi\psi})_{j\ell} = \frac{c}{\omega_\ell^2} \delta_{j\ell} \tag{35}$$

$$E(\delta_{\rm rms}^2)_{\rm ideal} = \sum_{j=1}^{n_s-n} \lambda_j = c \sum_{j=n+1}^{n_s} \frac{1}{\omega_j^2} \tag{36}$$

Similarly the matrix $C_{\delta\delta}$ is equal to $C_{\psi\psi}$ without its first n diagonal terms, so that from Eqs. (A3) and (35)

$$\sigma^2(\delta_{\rm rms}^2) = 2c^2\sigma_\varepsilon^4 \sum_{j=n+1}^{n_s} \frac{1}{\omega_j^4} \tag{C11}$$

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